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A
BRIEF ACCOUNT OF, and an INTRODUCTION TO,
EIGHT LECTURES,
IN THE
SCIENCE of MUSIC,

TO BE READ AS SOON AS

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The Radical Sources of MELODY and HARMONY,

Deduced from the rational Principles of the Philoſophy of Sounds, from Arithmetical Calculations, and from Geometrical Diviſions in the Conſtruction of Monochords, to aſcertain the different Scales of the ſeveral Genera of the *Greeks* and the Moderns; by a clear, a conciſe, and an intelligible Method, different from what has been attempted before.

BY

MARMADUKE OVEREND,
Organiſt of *Iſleworth*, *Middleſex*.

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INTRODUCTION.

THE love of knowledge and the deſire of improvement are deeply rooted in the minds of all; yet, from ſituation in life, the want of the opportunities of cheriſhing thoſe diſpoſitions: or the habits acquired by adopting, unexamined, the prejudices of thoſe we have a good opinion of: that deſire and that love often loſe their influence. To be enlightened, and to have the powers of the underſtanding extended, is not only the wiſh and the purſuit, but the privilege of the aſſiduous, both in the contemplative and in the active parts of life; even the indolent and the ignorant have no averſion to obtain the pleaſing reflections thoſe enjoy, who have trod the mazes of ſcience, or penetrated the receſſes of nature, could they find their way through the paths that would conduct them thither, with as little trouble as taking a nap in an eaſy chair.

Between theſe two claſſes of the induſtrious and the inactive; there are many worthy, good-intentioned, and generous, not deſtitute of abilities; who, for want of a proper plan in the early part of life, or through the neglect of thoſe who have had the care of their education, are not great proficients in the ſciences: yet, impelled by an amiable diſpoſition of mind, take a pleaſure in giving encouragement to ſuch of their contemporaries as have employed their time and attention to enlarge the bounds of knowledge, by ſeeking out and rewarding merit wherever they can find it. Such deſerve the greateſt praiſe; for if the pleaſure is great to conceive and to mature: to afford aid to bring forth, and to cheriſh, muſt impart to the patronizing mind, the moſt pleaſing ſenſations the poſſeſſors of wealth or power can receive.

The intention of this ſmall tract is to explain and aſſiſt the encouragers of, and the attendants on, a Courſe of Eight Lectures in the Science of Muſic, to be immediately be delivered as ſoon as Fifty Subſcribers have ſent in their names, when a room will provided, at the Weſt End of the Town, furniſhed with the proper apparatus of inſtruments, ſcales, and plans; to which not more than One Hundred Subſcribers, and none but thoſe that are ſuch, can be admitted, at One Guinea each for the Courſe.

As this is a performance on a plan never before attempted, and as, without ſome deſcription more than an advertiſement can impart, the deſign may not be underſtood; an account thereof is due to the public, as well as ſome preliminary remarks introductory thereto.

The obſcurity that prevails in the works of thoſe that have undertaken to explain the principles of the ſcience of muſic is ſo great, as to make many able profeſſors ſit down contented with a partial knowledge of it's radical principles; and, indeed, the many ſeeming abſurdities which thoſe writers works abound with, have diſcouraged the ingenious in their purſuits ſo much, as to make them believe that there is no poſſibility of aſſigning any reaſon for thoſe principles; or that, when attained, they are not worth the pains that have been taken.

The candid in the profeſſion that have been converſed with on this ſubject acknowledge the value of aſcertaining ſuch principles, but none knew the real advantages of ſuch a reſearch ſo well as the Author's late worthy friend and maſter Dr. *William Boyce*, whoſe knowledge and abilities juſtly entitled him to that merited ſituation he enjoyed many years as maſter of his Majeſty's band of muſicians; who having ſeen ſome of the calculations, was pleaſed not only to commend and approve of them, but gave every aſſiſtance a long-cemented friendſhip could hope for, to bring the ſame to perfection. By this means a complete theory of the ſounds in muſic has been eſtabliſhed, and the principles thereof fixed on a foundation ſo ample and extenſive, as to explain and alſo to define moſt minutely all the three genera of the diatonic, the chromatic, and the enharmonic of the ancient *Greeks*, with the different ſpecies belonging to thoſe genera; the ratios or proportions of which are determined by a ſeries of whole numbers, unincumbered with fractions, to eighteen places in all the ſcales. Dr. *Boyce* has verified in his moſt valuable manuſcript treatiſe of compoſition, purchaſed of his family, (wherein the niceties of modulation are diſcuſſed and explained) all the numbers to ſeventeen places in the diatonic intenſe only, extended by double flats and double ſharps, with their differences. Thoſe numbers have ſince been reduced to their leaſt terms in whole numbers, and brought to fifteen places, which will give all the diviſions in the preſent practice contained within the cycle of one octave. The proceſs by which theſe were attained is part of the intended courſe.

There will alſo be given, in the courſe of theſe Lectures, an explanation of the firſt principles of the diviſion of the true ſcale of muſic; it's conſtruction by major tones and limmas, or by major tones, minor tones, and major ſemitones; preceding which will be ſhewn, the difference of a major tone and a minor tone, with ſcales thereof, muſically, geometrically, and arithmetically conſtructed, wherein all the leſſer diviſions thereof will be ſeen, and all theſe compared with a ſcale of commas increaſing and decreaſing. Theſe will give the opportunity of ſhewing the uncertainties and the errors of authors ſince the time that *Ptolemy* wrote; many of thoſe authors miſapprehenſions of the *Greeks* will be pointed out, and this theory ſhewn to be eſtabliſhed on truth and reaſon.

The greater and leſſer ſyſtems of the *Greeks* will alſo be explained; the ſcala maxima ſhewn to be rational and of uſe, notwithſtanding it is inconcinnous. What the concinnous ſcale is: what was meant by the ſyſtem of a fourth or tetrachord: why the *Greeks* divided their ſcales into tetrachords: what is to be underſtood by their disjunct ſcales: their conjunct ſcales: by the ſynemmenons; the remiſſion of the acumen, and the intention of the gravitas: what acumen and gravitas mean: all this in a view not before noticed. The names of the *Greek* notes will alſo be explained, and found to convey different meanings than have been hitherto thought of by different commentators; ſome of whom have thought the term Νήτη to mean the loweſt, and others that it meant the higheſt, and many other new and curious obſervations on thoſe names. The ſeveral characters for the notes of their ſcales will be alſo claſſed, methodized, and reſtored, freed from the errors of *Meimbomus* in his tranſlations of the *Greek* authors, and in the diagrams of the ſcales of the modes, &c. found in *Alypius* and *Gaudentius*.

The errors or defects alſo of *Boetius*, *Glareanus*, *Galileo*, *Zarlino*, *Kircher*, *Rameau*, &c. will be ſhewn, and a particular notice taken of the Tentamen novæ Theoriae Muſicae, &c. of the learned and ingenious profeſſor *Euler*.

The ratios of *Ptolemy* and the other *Greeks* will alſo be conſidered, as reſtored by Dr. *Wallis*, and compared with the ratios of the above ſcales of whole numbers, reduced to ſexageſſimals to make their compariſons eaſy, ſhewing how much they differ, and where erroneous corrected.

The reaſon will be explained why fretted and keyed inſtruments diſagree with the voice, or with the ſounds produced by a fine performer on the violin; the reaſon of the temperament on the harpſichord and organ; a propoſal for the beſt way of adjuſting thoſe inſtruments; hints for an improved harpſichord, to be played upon with as much eaſe as the preſent inſtruments with keys are, and yet to have the ſharps and the flats diſtinct notes, and not to ſubſtitute a very diſcordant ſound for the true one, as at preſent; and this in a different way from the late Dr. *Smith's* inſtrument, without ſhifting the ſtops.

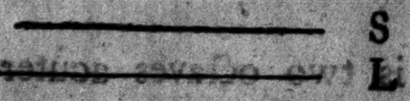
More will be added in the courſe; at preſent thefollowing pages ſhew the manner of finding the ratios of the ſounds by ſtrings and by their numbers or menſurate quantity, in a way different from what theſe operations have been performed heretofore, the which, it is hoped, will be found an uſeful aſſiſtant in going through the Lectures with intelligence and ſatisfaction.

A Table of Intervals in Muſic, with their Ratios.

The radical ſound or uniſon	1 or $\frac{1}{1}$	A major third	$\frac{5}{4}$
A major comma or ſimply comma	$\frac{81}{80}$	A flat fourth	$\frac{32}{25}$
The enharmonic dieſis	$\frac{128}{125}$	A fourth	$\frac{4}{3}$
A ſemitone ſubminimum	$\frac{250}{243}$	A ſharp fourth or tritone	$\frac{45}{32}$
A ſemitone minimum	$\frac{648}{625}$	A flat fifth or the ſemideapente	$\frac{64}{45}$
A ſemitone minor	$\frac{25}{24}$	A fifth	$\frac{3}{2}$
A leimma	$\frac{256}{243}$	A ſharp fifth	$\frac{25}{16}$
A ſemitone medius	$\frac{135}{128}$	A minor ſixth	$\frac{8}{5}$
A major ſemitone	$\frac{16}{15}$	A major ſixth	$\frac{5}{3}$
An apotome	$\frac{2187}{2048}$	A flat ſeventh	$\frac{128}{75}$
A ſemitone maximum	$\frac{27}{25}$	An extreme ſharp ſixth	$\frac{225}{128}$
A minor tone	$\frac{10}{9}$	A leſſer ſeventh, being the major tone inverted	$\frac{16}{9}$
A major tone	$\frac{9}{8}$	A greater ſeventh, being the minor tone inverted	$\frac{9}{5}$
A deficient third	$\frac{32}{27}$	A ſharp ſeventh	$\frac{15}{8}$
A minor third	$\frac{6}{5}$	An eighth or octave	$\frac{2}{1}$

A ſhort and eaſy Method of finding the Ratios of Muſical Intervals by Operations, performed on Lines or Strings.

1. IN the following two lines, S. L. the eye inſtantly diſcovers a difference in their length.



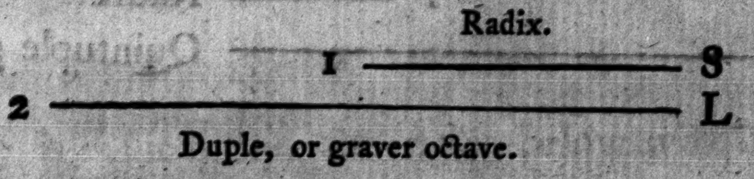
2. What that difference is may be exactly expreſſed by numbers, if meaſured with a pair of dividers; by which, if their lengths are compared, their ratios will be found to be according as the compariſon is made, either in duple or in ſubduple ratio.

3. If S is firſt given to know the ratio of L to it, then S is named the radix, and L is to S in duple ratio.

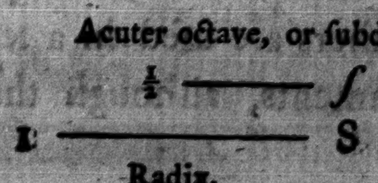
4. If L is firſt given to find the ratio of S to it, then L is made the radix, and S is to L in ſubduple ratio.

5. If theſe two lines were two muſical ſtrings of the ſame thickneſs or diameter, S of the length of one foot, and L of the length of two feet, and ſtretched by equal weights or tenſions, the interval in muſic they would make would be, when ſtruck and compared, the interval of an octave, eſtimated according to their order of ſounding, 1 : 2, or 2 : 1.

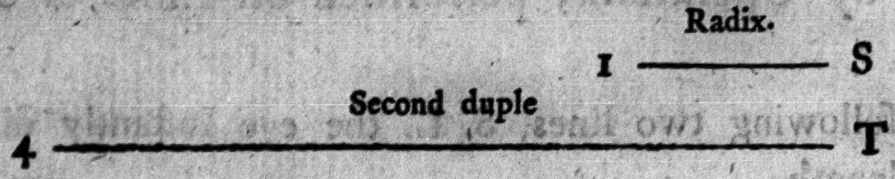
6. If the ſound of 1, S, is made the radix, to which the ſound of 2 L is compared, that interval is the graver octave, and it's ratio $\frac{2}{1}$ is the duple, or 1 multiplied by 2.



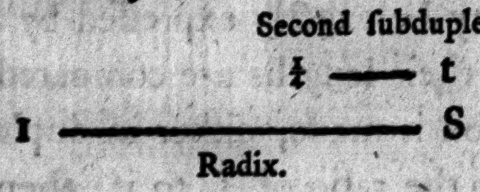
7. If 1, S, is made the radix, to which the ſound of $\frac{1}{2}$ ſ is compared, that interval is the acuter octave to S, and it's ratio $\frac{1}{2}$ is the ſubduple, or 1 divided by 2.



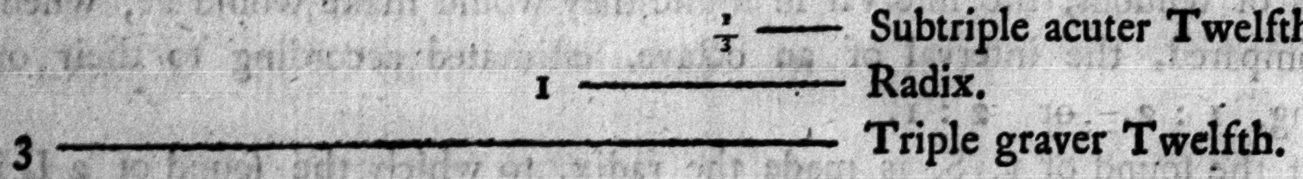
8. If two octaves, graver than the radix, are required, the firſt duple muſt be dupled again; this is named the 2d duple; it's ratio is alſo duple to the firſt duple: therefore, making 1 S again the radix, the 2d octave graver is 4 T, or $\frac{4}{1}$, being 1 multiplied by 4.

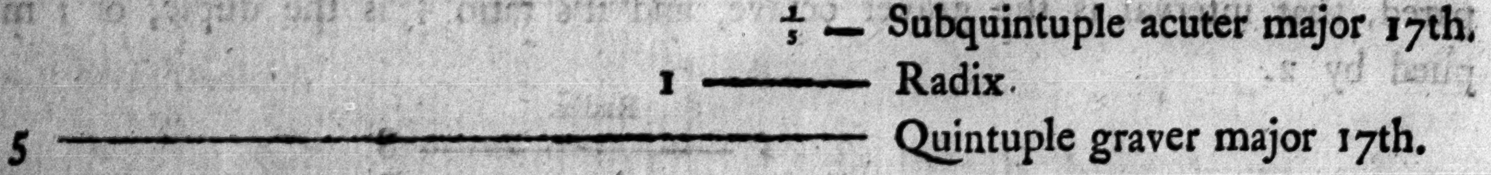


9. In like manner $\frac{1}{4}$ t is two octaves acuter than the radix 1 S, and is the 2d ſubduple thereof, $\frac{1}{4}$ or 1 divided by 4.



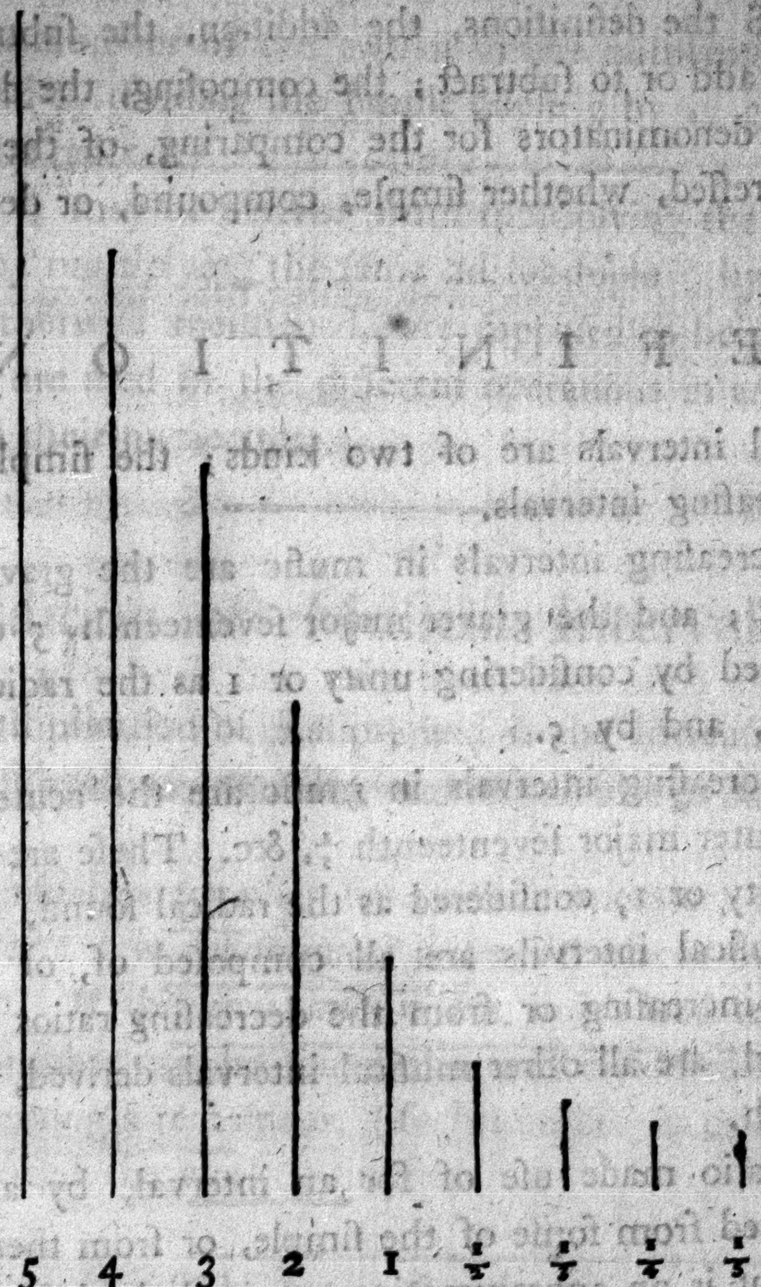
10. What has been done with the duples and ſubduples of 1 or unity, may be done with the triples and ſubtriples, the quintuples and the ſubquintuples thereof to infinity increaſing, and to ſeeming infinity, though certainly finite when decreaſing.





11. No other increaſes of the lengths of ſtrings or decreaſes, but what ariſe from theſe or their various and infinite combinations, can produce ſounds uſeful in muſical compoſition, though many others are uſed in a tempered ſcale, and as ſuch are borne with from imperfect inſtruments, although they would be condemned in a Singer or a Violiniſt, &c.

12. The above Lines with their Ratios collected into one View.



13. If the $\frac{1}{5}$ is made unity, the ratios below will be proportionate.

25, 20, 15, 10, 5, $\frac{5}{2}$, $\frac{5}{3}$, $\frac{5}{4}$, 1.

14. If 5 is made unity then theſe are like proportionals.

1, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{1}{25}$.

15. If either of the above are reduced to integers in their leaſt terms, they will be

300. 240. 180. 120. 60. 30. 20. 15. 12.

MUSICAL ARITHMETIC

16. CONTAINS the definitions, the addition, the ſubtraction, the reduction, in order to add or to ſubtract; the compoſing, the de-compoſing, and the reducing to common denominators for the comparing, of the ratios by which muſical intervals are expreſſed, whether ſimple, compound, or decompound.

DEFINITIONS.

17. Simple muſical intervals are of two kinds; the ſimple increaſing intervals, and the ſimple decreaſing intervals.

18. The ſimple increaſing intervals in muſic are the graver octave 2 or $\frac{2}{1}$; the graver twelfth 3 or $\frac{3}{1}$; and the graver major ſeventeenth 5 or $\frac{5}{1}$, &c. Theſe are integral ratios produced by conſidering unity or 1 as the radical ſound, and multiplying it by 2, by 3, and by 5.

19. The ſimple decreaſing intervals in muſic are the acuter octave $\frac{1}{2}$, the acuter twelfth $\frac{1}{3}$, and the acuter major ſeventeenth $\frac{1}{5}$, &c. Theſe are fractional ratios found by the diviſions of unity or 1, conſidered as the radical ſound, by 2, by 3, and by 5.

20. The other muſical intervals are all compoſed of, or de-compoſed by, the ſimple; for from the increaſing or from the decreaſing ratios of the ſimple muſical intervals above defined, are all other muſical intervals derived, by Addition or Subtraction, or from both.

21. Every other ratio made uſe of for an interval, by any author heretofore, which cannot be derived from ſome of the ſimple, or from their combinations, is not a muſical number, but is an approximation, or is derived from a tempered ſyſtem, and ſubſtituted for the true ratio.

22. Theſe ratios always conſiſt of two numbers, one of which is placed over the undermoſt, and named the numerator; the other number placed under the uppermoſt is termed the denominator.

23. The denominator ſhews how many parts any quantity is divided or ſuppoſed to be divided into, and the numerator how many of ſuch diviſions the ratio conſiſts of; or the numerator may be conſidered as a quantity, and the denominator will expreſs what part thereof.

24. When the greateſt number is in the numerator, as $\frac{5}{4}$, it is a ratio greater than unity, formed from the ſimple quintuple 5, divided by 4; this gives the increaſed major third below the radix 1.

25. But if the leſſer number of the two is in the numerator, as $\frac{3}{4}$, it is a ratio leſſer than unity, found by dividing the ſimple triple 3 by 4, giving thereby the decreaſed fourth above the radix 1.

26. The ſame ratio $\frac{5}{4}$ may be derived from multiplying the 2d ſubduple $\frac{1}{4}$ by 5; and alſo the ratio $\frac{3}{4}$, by multiplying the ſame 2d ſubduple $\frac{1}{4}$ by 3.

27. All ratios not otherwiſe mentioned, are ſuppoſed to be quantities greater than unity; and when they are uſed in the different operations in arithmetic, are to have their largeſt number in their numerators.

Addition of Muſical Intervals

IS performed by Multiplication of Ratios, and is the increaſing of muſical ratios, which may be conſidered as extending a muſical ſtring; to perform which this is the Rule:

28. Multiply the numerators together for a new numerator, and alſo multiply the denominators together for a new denominator; the firſt placed over the laſt, gives the required interval's ratio. If this new ratio is not in it's leaſt terms, it muſt be reduced thereto, if any number can be found that will divide the numerator and the denominator without leaving a remainder, (ſuch number is called a common meaſurer) and when ſo reduced the work is finiſhed.

To add a fifth $\frac{3}{2}$ to a fourth $\frac{4}{3}$.

$$\overset{\text{by}}{\underset{\text{by}}{\frac{3}{2} \times}} \overset{\text{make}}{\underset{\text{make}}{\frac{4}{3}}} = \frac{12}{6} \text{ This divide by } 6\Big)\frac{12}{6}\Big(\text{make } \overset{\text{The increaſing octave}}{\underset{\text{Or an octave graver}}{\frac{2}{1}}}$$

29. Or the two ratios may be firſt reduced, if any number can be found that will divide the numerator of the one, and the denominator of the other, without any remainder; and alſo if any number ſhall be found in the denominator of the one, and in the numerator of the other, they may be both cancelled or ſcratched acroſs by a

diagonal

diagonal ſtroke; and the remaining figures after theſe reductions, multiplied together, will give the required ſum of the interval ſought.

Thus in the ſame example, finding that 2 in the firſt will divide the numerator 4 in the ſecond ratio twice, place that quotient 2 over the 4, and cancel the 4; and as it will divide the denominator of the firſt ratio once, it will be unneceſſary to take notice of that 1, only cancel the 2, and the work will be thus:

$$\frac{3}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{3}$$

Then as in the other two places there are a 3 above, and alſo a 3 below, cancel them both, and the 2 above the ſecond ratio is the required ſum, under which an unite muſt be placed when all the figures in the denominators are cancelled. This gives the duple as before, and the whole ſtands thus:

$$\frac{\cancel{3}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}} = \frac{2}{1}$$ **The graver Octave.**

Another Example.

30. Add to a fourth $\frac{4}{3}$ a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$

$$\frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} = \frac{2}{1}$$

Here 4 in the firſt and ſecond ratios, and 5 in the ſecond and the third ratios, cancel each the other; alſo 3 will divide 6 twice; put the quotient 2 over the 6, then cancel that 6 and alſo the 3 in the firſt ratio; the 2 then only being left above, ſhews theſe to make alſo the graver octave, as below.

$$\frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{5}} = \frac{2}{1}$$

More

More Examples of Addition.

31. Add a fourth $\frac{4}{3}$ a fourth $\frac{4}{3}$ and a major tone $\frac{9}{8}$ together.

$$\frac{\cancel{4}}{\cancel{3}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}} \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\underset{\cancel{4}}{\cancel{8}}} = \frac{2}{1} \text{ Octave.}$$

32. To add a Comma to a Comma.

$$\frac{81}{80} \times \frac{81}{80} = \frac{6561}{6400} = \text{2 Commas.}$$

33. To add two major tones together.

$$\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$$ This is the ſuperfluous third in the ſcala maxima.

34. To add a major tone $\frac{9}{8}$ a minor tone $\frac{10}{9}$ and a major ſemitone $\frac{16}{15}$ together.

$$\frac{\cancel{9}}{\cancel{8}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{9}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{4}{3} \text{ A fourth.}$$

Here 9 in the one part cancels the 9 in the other, 8 divides 16 twice, the 2 put over the 16 and cancel both; 10 gives 2, and 15 gives 3 if divided by 5; cancel 10 and 15, then 2 multiplied by 2 produces 4 for the new numerator, and 3 in the denominator put under the 4 makes the ratio of a fourth, as above.

35. Two fourths added is

$$\frac{4}{3} \times \frac{4}{3} = \frac{16}{9} \text{ a leſſer ſeventh.}$$

This leſſer ſeventh $\frac{16}{9}$ added to a major tone $\frac{9}{8}$ make $\frac{2}{1}$ the octave.

Subtraction of Muſical Intervals

IS performed by diviſion of ratios, and may be conſidered as the diminiſhing of a muſical ſtring in proportion to the quantity of the propoſed ratio, no alteration being made in it's tenſion; to perform which this is the Rule:

36. Multiply the numerator of the interval to be ſubtracted into the denominator of the interval from whence it is to be taken, and the product will be a new denominator; then multiply the denominator of the interval to be ſubtracted into the numerator of the interval from whence it is to be taken, and place the product over the firſt found product for a new numerator. The interval expreſſed by this new ratio will be the ſought difference, if it is in the loweſt terms, elſe it muſt be reduced by a common meaſurer as before.

37. Take a fourth $\frac{4}{3}$ from an octave $\frac{2}{1}$.

$\frac{4}{3}$ from) $\frac{2}{1}$ (leaves $\frac{6}{4}$ reduced to $\frac{3}{2}$ A fifth.

Here 4 in the firſt, multiplied by 1 in the ſecond ratio, make 4 for the new denominator; and 3 in the firſt, multiplied by 2 in the ſecond ratio, make 6 for the new numerator; that ratio divided in both parts by 2, gives the reduced ratio of a fifth, for the required difference.

38. The ſame may be reduced firſt, if a number can be found that will divide both the numerators without any remainder, or both the denominators without any remainder. Thus 2 will divide 4 in the firſt ratio, twice; place the quotient 2 over the 4, and cancel the 4; cancel alſo the 2 in the ſecond ratio, as 2 is once contained in it; then, as there is no figure to multiply 3 with, carry it up to the numerator of the difference, and multiply the 2 by 1 for the denominator of that difference, and the operation will ſtand as below, and ſhew the difference to be a fifth.

$\frac{\overset{2}{\cancel{4}}}{3}$ from) $\frac{\cancel{2}}{1}$ (leaves $\frac{3}{2}$ = A fifth.

39. Take a fourth $\frac{4}{3}$ and a major third $\frac{5}{4}$ from an octave $\frac{2}{1}$

$\frac{\cancel{4}}{3} + \frac{5}{\cancel{4}}$ from $)\ \frac{2}{1}\ ($ leave $\frac{6}{5}$ A minor third.

Here the firſt two ratios being to be added together before they are ſubtracted, are before that operation reduced by cancelling the 4 in each, then 5 is multiplied by 1 to make the denominator, and the 3 by 2 to make the numerator.

40. Take a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$ from an octave $\frac{2}{1}$

$\frac{\cancel{5}}{\underset{2}{\cancel{4}}} + \frac{\overset{3}{\cancel{6}}}{\cancel{5}}$ from $)\ \frac{2}{1}\ ($ leave $\frac{4}{3}$ A fourth.

Here 5 being found in the numerator and the denominator of the two intervals to be added together before their ſum is to be ſubtracted, both are cancelled; then 2 will divide the 4 and the 6 without any remainder: theſe being cancelled alſo, and their quotients multiplied croſs-ways, 3 with 1, and 2 with 2, give $\frac{4}{3}$, or a fourth, for the difference ſought.

41. Take a major tone $\frac{9}{8}$ from a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$

$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}}$ from $)\ \frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{4}}} + \frac{\overset{\cancel{3}}{\cancel{6}}}{\cancel{5}}\ ($ leaves $\frac{4}{3}$ A fourth.

Here each 5, in the ratios to be added, is firſt cancelled; then finding that 4 and 6 in the upper and in the lower part of thoſe two ratios may be divided by 2, they are brought to 2 below and 3 above, and the 4 and 6 are cancelled alſo; then 9 in the ratio to be ſubtracted, and 3 in the laſt of the ratios from whence it is taken being both diviſable by 3, put down 3 over the 9, and cancel that 9 and the 3 in the laſt ratio: after that 8 and 2 are diviſable by 2, put down the 4 under 8, and cancel that 8 and the 2. After theſe operations, there being nothing to multiply 3 with in the lower parts of the two laſt ratios, becauſe they are all cancelled in both, place that 3 beyond all for a new denominator, and in like manner place the 4 over it for a new numerator, and that ratio is a fourth, the required difference.

The Reduction of Ratios, in order to add or to ſubtract, depends on this ſhort Theorem.

42. IF ratios are multiplied in their numerators and in their denominators by any number, or if any number will divide their numerators and their denominators without a remainder, they will be found after either or both thoſe operations to be in the ſame proportion as they were before, but in higher or lower terms.

43. Thus $\frac{1}{2}$ multiplied by 3 in both the numerator and in the denominator will make $\frac{3}{6}$, which is the ſame ſubduple in higher terms.

44. Alſo $\frac{24}{16}$, divided in both the numerator and in the denominator by 8, gives $\frac{3}{2}$, which is the ſame quantity in the leaſt terms.

45. Further 6 : 5 : 3, multiplied by 2, make 12 : 10 : 6, equal to the above.

46. Alſo 16 : 18 : 20, divided by 2, give 8 : 9 : 10, in their leaſt terms.

47. Likewiſe 45 : 50 : 60, divided by 5, give 9 : 10 : 12, which are proportionally the ſame as thoſe quantities were before ſuch diviſion.

The Reduction, in order to compare two or more Ratios together, is thus performed.

48. MULTIPLY the denominators of both the ratios together, if there are only two ratios: or multiply the denominators of all the ratios together, if there are more than two ratios, for a common denominator; then multiply each of the numerators ſeparately by all the denominators of each other ratio but it's own: theſe laſt products, placed ſeverally over the common denominator firſt found, will give new ratios equal to the firſt given ratios, and their compariſons may thereby be clearly ſeen. If after theſe operations they can be reduced to lower terms by any number that will divide all the numerators and all the denominators, without any remainder, the reſult will give other ratios in the ſame proportion.

EXAMPLE I.

49. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to common denominators.

The denominator of the firſt fraction 2, multiplied by the denominator of the ſecond fraction 3, will produce the common denominator to both, 6.

Then, the numerator of the firſt fraction 1, multiplied by the denominator of the ſecond fraction 3, will produce the new numerator of the firſt fraction 3.

Further, the numerator of the ſecond fraction 2, multiplied by the denominator of the firſt fraction 2, will produce the new numerator of the ſecond fraction 4.

Theſe new numerators being each wrote over the common denominator, will make theſe:

$$\frac{3}{6} \text{ and } \frac{4}{6} \text{ equal to } \frac{1}{2} \text{ and } \frac{2}{3}$$

EXAMPLE II.

50. Let $\frac{9}{8}$ $\frac{6}{5}$ $\frac{5}{4}$ and $\frac{4}{3}$ be compared by reduction.

Firſt denom.	8	firſt num.	9	ſecond num.	6	third num.	5	fourth num.	4
Second den.	5		5		8		8		8
	40		45		48		40		32
Third den.	4		4		4		5		5
	160		180		192		200		160
Fourth den.	3		3		3		3		4
Com. den.	480		540		576		600		640

Theſe placed in order over the common denominator, give

$$\frac{540}{480} \quad \frac{576}{480} \quad \frac{600}{480} \quad \frac{640}{480}$$

$$\text{Equal to } \frac{9}{8} \quad \frac{6}{5} \quad \frac{5}{4} \quad \frac{4}{3}$$

51. Theſe may be all reduced into their leaſt terms by dividing each member of the whole by 4, which, when done and compared with unity or $\frac{1}{1}$, the radix, (by multiplying it in the numerator and in the denominator by the reduced common denominator) give the under-placed ratios with their intervals or differences between each other.

Radix	$\frac{120}{120}$	tone major	$\frac{135}{120}$	ſemitone major	$\frac{144}{120}$	ſemitone minor	$\frac{150}{120}$	ſemitone major	$\frac{160}{120}$
Sounds	G		F		E		E flat		D

52. By ſimilar operations are ratios alſo reduced to whole numbers, for if the numerators of the laſt reduced ratios are taken without their common denominator, they will give a ſeries of whole numbers in the ſame proportion as the ratios at firſt were; the firſt term of which ſeries will be that common denominator for the radix.

Radix 120 : 135 : 144 : 150 : 160

53. Theſe may alſo be eaſily converted into numbers with decimal parts, or decimals of 1, 2, 3, &c. places, thus:

12.0 : 13.5 : 14.4 : 15.0 : 16.0 mixed
1.20 : 1.35 : 1.44 : 1.50 : 1.60 mixed
.120 : .135 : .144 : .150 : .160 decimals
.0120 : .0135 : .0144 : .0150 : .0160 decimals

Each of theſe are a ſeries alike proportional, being made ſeverally a tenth, an hundredth, a thouſandth, and a ten thouſandth leſs than the firſt ſeries of whole numbers.

54. If either unity or a whole number is made uſe of in any of the above operations, it muſt be expreſſed as a ratio by putting 1 under it thus, as $\frac{1}{1}$ or $\frac{4}{1}$, for 1 and 4, &c.

COROLLARIES.

55. EVERY ratio having 1 or unity for it's denominator is a whole number, as $\frac{1}{1}$ is 1, $\frac{2}{1}$ is 2, $\frac{3}{1}$ is 3, &c.

56. Every whole number is always ſuppoſed to have 1 or unity under it for it's denominator, to aſſiſt the underſtanding in having a clear idea thereof, thus, 6 is $\frac{6}{1}$, or ſix quantities, one of which ſuch quantities is conceived and determined in the mind, as ſix ſhillings, ſix yards, or ſix days, &c.

57. Every ratio whoſe numerator can be exactly divided without leaving any remainder by its denominator, and that being more than 1 or unity, is alſo a whole number wrote as a ratio, but not in it's loweſt terms. *Such are improperly named by arithmeticians, improper fractions, for the term improper muſt convey to the mind a falſe or confuſed idea.*

58. All other ratios having their numerators greater than their denominators, being not exactly diviſable thereby, and their denominators greater than 1 or unity, are mixed numbers or quantities greater than a whole number, being an unit or a whole number and a fraction, as $\frac{6}{5}$ gives 1 $\frac{1}{5}$; $\frac{9}{2}$ is 4 $\frac{1}{2}$; and $\frac{9}{4}$ is 2 $\frac{1}{4}$. *Theſe are alſo improperly named improper fractions.*

59. Any ratio having in it's numerator 1 or unity, and in it's denominator ſome number greater than unity, but which cannot be divided by any other number than unity, is a ſimple fraction, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, &c.

60. If the denominator of any fractional ratio, having unity for it's numerator, can be divided by any other number beſides unity, it is a decompound fraction, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{9}$, &c. The firſt may be divided by 2, the ſecond by 2 and by 3, and the third by 3.

61. Every ratio in it's leaſt terms having a number greater than unity in it's numerator, and it's denominator greater than the numerator, is either a compound fraction, or elſe a decompound fraction, according to the manner of it's production; it may alſo be a compound and a decompound at the ſame time.

62. The compound fraction $\frac{3}{5}$ is made up of three, fifth parts of an unit, if $\frac{1}{5}$ is given to multiply it by 3.

63. The ſame $\frac{3}{5}$ may be termed a decompound fraction, if 3 or $\frac{3}{1}$ is given to be divided by 5; it is then in that caſe decompounded, or underſtood as the fifth part of the quantity 3.

64. Further, the ſame $\frac{3}{5}$ may be both a compound and a decompound fraction, if found by having $\frac{1}{1}$ given to multiply it by 3, and afterwards to divide that product by 5, this is an unit firſt encreaſed three times and afterwards decreaſed a fifth part.

65. The idea of compound, is an increaſe of quantity.

66. The idea of decompound is a decreaſe of quantity.

67. Hence the fraction of a fraction, as the $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{5}$ is a decompound fraction, and not a compound fraction, as has always been heretofore taught; for the $\frac{1}{5}$ of an unit is thereby made the $\frac{1}{40}$ part thereof.

68. If a ratio or a fraction is to be multiplied by a number, and that number will evenly divide the denominator of ſuch ratio, the quotient of ſuch diviſion placed under the numerator will give a ratio equal to the firſt ratio, if it's numerator was multiplied by ſuch number, and afterwards reduced to its leaſt terms, which is the manner of doing it, if the number will not evenly divide the denominator. Thus, $\frac{2}{9}$ multiplied by 3 makes $\frac{2}{3}$, whether the 2 is multiplied by 3, or the 9 divided by 3; in the firſt caſe, there will be found $\frac{6}{9}$, which reduced is $\frac{2}{3}$, as before.

69. If any fraction or any ratio is to be divided by a number, and that number will evenly divide the numerator of ſuch ratio or fraction, the quotient of ſuch diviſion placed over the denominator will give a ratio equal to the firſt ratio, if it's denominator was multiplied by ſuch number and afterwards reduced to it's leaſt terms, which muſt be done if that number will not evenly divide the numerator. Thus, $\frac{8}{9}$ divided by 4 gives $\frac{2}{9}$; whether 8 is divided by 4 firſt, or whether 9 is multiplied by 4, making $\frac{8}{36}$, and afterwards reduced to it's leaſt terms, it gives $\frac{2}{9}$, as above.

70. All numerators are real multipliers or increaſers, as $\frac{3}{\ }$ or $\frac{5}{\ }$, &c.

71. All denominators are real diviſors or decreaſers, as $\frac{\ }{2}$ or $\frac{\ }{3}$, &c.

What is here delivered has produced eaſy ſolutions to a vaſt number of abſtruſe and controverted queſtions on the diviſions of the ſcales of muſic; and ſeveral problems have and may again be wrought in a few minutes, by the aſſiſtance of ſome harmonical tables conſtructed thereby, as would in the common way of calculation require ſeveral years for their performance, even if no error ſhould occur in the operation that way, which it is hardly poſſible to expect, for frequently the numbers are 60, 100, or more places, before their reduction to their leaſt terms.

As an inſtance of what is here advanced, and to ſhew the brevity of ſuch method, are given the following ratios, which are the quantities that are contained in the compaſs of one octave of the ſcale of ſounds belonging to the ſpecies of the diatonic intenſe, in both the acumen and in the gravitas, with the reduced extenſions to double flats and double ſharps; all which the late Dr. *Boyce* verified from four radical numbers firſt aſcertained and imparted to him for that purpoſe.

Within the extremes of ſuch an octave are contained all theſe differences between it's ſeveral component parts.

The number	of the ratios,	and the nominal quantities of each.
Four of	$\frac{10737418240}{10460353203}$	An enharmonic dieſes and a minor reſidual.
Two of	$\frac{70368744177664}{68630377364883}$	Two minor commas and a minor reſidual.
Six of	$\frac{858993459200}{847288609443}$	A minor comma and a minor reſidual.
Sixteen of	$\frac{81}{80}$	A major comma.
Nineteen of	$\frac{2048}{2025}$	a minor comma. And
Thirty-eight of	$\frac{32805}{32768}$	A ſchiſma.

The above make 85 differences between 86 ſounds, which are the number of ſubdiviſions in that octave. When they are all ſummed up and reduced, they make $\frac{2}{1}$.

They have been, and at any time may again be added, by any perſon only acquainted with the firſt two rules of arithmetic, in leſs than ten minutes.

A minor reſidual is $\frac{10485760000}{10460353203}$

The pleaſure of having made this diſcovery is a ſufficient reward for the application and labour beſtowed on the reſearch, even if no other advantage ſhould be derived from the ſame.

CONCLUSION.

CONCLUSION.

EVERY thinking enquirer after improvement, to whoſe hands this may happen to make it's way, muſt perceive the great advantages to be derived from an inveſtigation of theſe truths. For, whoſo ſhall be able to claſs and arrange methodically and clearly; to mark down and to point out, the diviſions and the diſtinctions of all the ſounds in melody and in harmony, belonging to the ſcales of the different ſpecies of all the genera, in muſic, accurately and alſo evidently unto the ſenſes and unto the reaſon; to aſcertain the component parts thereof, their ratios and their connections with their differences exactly and minutely; and by theſe means to fix in the mind indiſputably and indubitably, that beautiful order in which the principles of this DIVINE SCIENCE have always exiſted, ſtill remain unchanged, and eternally ſhall endure— Muſt have as much advantage above another not ſo informed, as one voyager with an exact chart and compaſs muſt have over another aſſiſted by repeated trials and experience only, notwithſtanding the knowledge of the latter may be thought and likewiſe be found to be ſufficient.

THE END.

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